Vector Space – Salient Points

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1) Vector space is a composite object:

a. Vector Space
   i. Scalar Field **F**
   ii. Vector Set **V**
   iii. Operations
      1. Addition of two vector objects
      2. Multiplication by scalar

b. Example $\mathbb{R}^n$ or $\mathbb{C}^n$ with typical established laws for addition and scalar multiplication

2) Rules for vector addition operation:

For $u, v, w \in \boldsymbol{V}$

a. Commutative: $u + v = v + u$
b. Associative: $(u + v) + w = u + (v + w)$
c. Zero vector 0: $u + 0 = u$
d. Unique vector $-u$: $u + (-u) = 0$

3) Rules for scalar multiplication operation:

a. $1u = u$
b. $(\alpha_1 \alpha_2)u = \alpha_1(\alpha_2 u)$
c. $\alpha(u + v) = \alpha u + \alpha v$
d. $(\alpha_1 + \alpha_2)u = \alpha_1 u + \alpha_2 u$

4) Inner product operation rules:

For $u, v, w \in \boldsymbol{V}$, and for a given rule for inner product of two vector objects u and v represented by $\langle u, v \rangle$

a. Conjugate symmetry -

$$\langle u, v \rangle = \overline{\langle v, u \rangle}$$

b. Linearity –
   i. $\langle \alpha u, v \rangle = \alpha \langle u, v \rangle$
   ii. $\langle u + v,\ w \rangle = \langle u, w \rangle + \langle v, w \rangle$

c. Positive definiteness

$$\langle u, u \rangle \geq 0 \ \& \ \langle u, u \rangle = 0 \ \Rightarrow u = 0$$