Binomial (n, p) is re-written as following:

$$
\begin{aligned}
P\{X = i\} &= \binom{n}{i} p^i (1-p)^{n-i} \\
&= \left(\frac{n(n-1)(n-2)...(n-i+1)}{i!}\right)\left(\frac{\lambda}{n}\right)^i \left(1-\frac{\lambda}{n}\right)^{n-i} \\
&= \left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\cdots\left(\frac{n-i+1}{n}\right)\left(\frac{\lambda^i}{i!}\right)\left(1-\frac{\lambda}{n}\right)^n \left(1-\frac{\lambda}{n}\right)^{-i} \qquad i = 0, 1, ..., n.
\end{aligned}
$$

For very large n and moderate λ we can approximate

$$
\left(1-\frac{\lambda}{n}\right)^n \approx e^{-\lambda}; \qquad \frac{n-1}{n} = \frac{n-2}{n} = \frac{n-3}{n} ... = \frac{n-i+1}{n} \approx 1; \qquad \left(1-\frac{\lambda}{n}\right)^i \approx 1
$$

Hence,

$$
P\{X = i\} = e^{-\lambda}\frac{\lambda^i}{i!}
$$

which is Poisson's distribution.