# Vector Calculus Final Exam

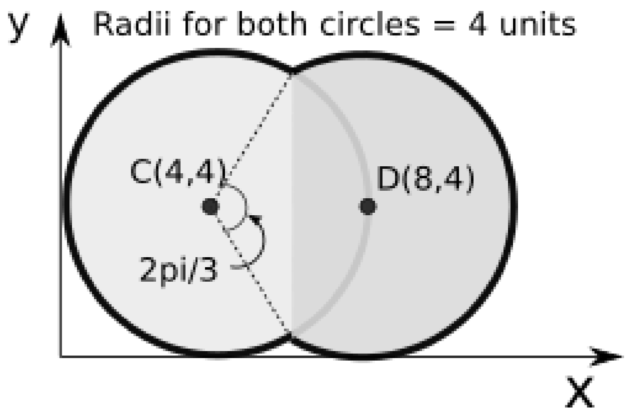
**Engineering Mathematics for Advanced Studies**
IIT Dharwad
Autumn 2019

Date - Monday 25th Oct. 2019
Time - 2 Hours (2:00pm -4:00pm)
Maximum score - 60; Minimum score - 0
Rule for absentee - Minimum 30% penalty; discuss reasons absense in person to get a chance for re-test.

1. Worked out solutions are must for most of the problems.
2. distance of a point P from line AB is $d = \frac{\|\vec{v}\times\vec{w}\|}{\|\vec{v}\|}$ where $\vec{v}$ is vector along AB and $\vec{w} = \vec{r}_P - \vec{r}_A$ where r is position vector
3. Formulae for standard solutions will be provided. One A4 sized cheat sheet is allowed. If vector $\vec{f} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$ (note - subscript does not mean derivative here) and scalar function $\phi = \phi(x, y, z)$ is given
   (a) $\vec{\nabla}\cdot\vec{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$
   (b) $\vec{\nabla}\phi = \frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}$
   (c) $\vec{\nabla}\times\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$
4. $arcLength = \int_a^b \sqrt{r'.r'}dt$
5. If $\vec{f}(x, y) = P(x, y)\hat{i} + Q(x, y)\hat{j}$ is the vector field and a curve C defines boundary of a region D:
$$\int_C (Pdx + Qdy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA$$
6. If $\vec{f}(x, y, z)$ is a vector function
$$\int\int\int_V \vec{\nabla}\cdot\vec{f}dV = \int\int_S \vec{f}\cdot\hat{n}dA$$
7. Please ensure to write Question number in a box as a heading to the upcoming answer on suppliments e.g. $\boxed{\text{Question 1}}$

# QUESTIONS

1. Find all local maxima and minima of $f(x, y) = (x-2)^4 + (x-2y)^2$ (marks 4)
2. Using double integral estimate the area of the following shaded area.
   (Hint: Leverage symmetry. Use standard formula for area estimation wherever possible. (marks 8)



3. If point P(2,3,1) and point Q(1,2,2) are given and it is needed to find point R such that point Q **divides** the line segment PR in proportion PQ:QR=5:4.

    (a) what are the coordinates of R? (marks 2)

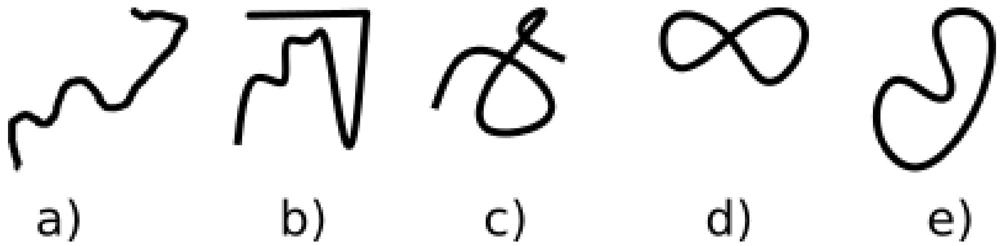
    (b) If point S(1,2,3) is given, what is the angle subtended at point S by line segments PS and QS? (marks 2)

4. For the diagram given in Question 2 above, please evaluate the line integral $\oint \vec{f}.d\vec{r}$ along the entire boundary curve if vector $\vec{f} = -2y\hat{\imath} + x\hat{\jmath}$. (marks 4)

5. There is rope-way traffic movement of cargo along the edges of a triangle formed by connecting three peaks in a Hilly region. Say points A, B and C with position vectors $\vec{a}$,$\vec{b}$,$\vec{c}$, respectively form this triangle in eucledian space. It necessary to set-up an observatory at Hill D such that it can monitor cargo along all three possible paths i.e. AB, BC, and CA. Here, monitoring involves only detecting presence of transmitter mounted on the cargo box in-transition using a receiver in the located in observatory D. Even single instance of presence detection anywhere during the transition is good enough. It is assumed that once cargo box leaves a station it does not return back i.e. it always reaches other end of the rope way. Presence is detected only if the tranmitter is within the range of the receiver. Find the minimum range needed for transmitter-receiver system to be deployed if A(1,0,2), B(0,2,3), C(3,3,0) and D(1,1,1). (marks 8)

6. Give an expression for the distance travelled by an ant if she is taking a spiral path from a funnel tip at the bottom at time t=0 and moves upwards. You may assume funnel surface to be a upside down circular cone with **half-angle** 30 degrees. Ant is completing each turn arround in the same time and upward movement per turn remains the same.
   Note - Please provide list of variables/symbols and their quick description just after the final expression for the distance travelled as a function of the time. (marks 8)

7. List all simple curves in the following (marks All or None) (marks 6)



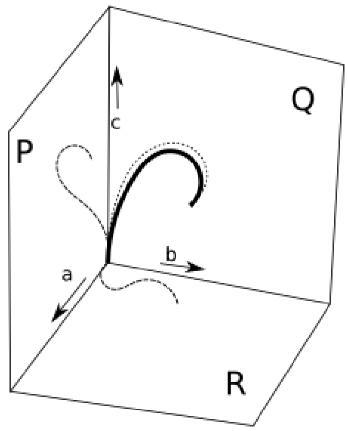

8. Find tangent plane to a surface $x^2 + 2y^2 + z^2 = 13$ at (2,2,-1). (marks 4)

9. Is the 3D volume defined by the interior of a normal disposable paper cup is a simply connected domain ? (marks 2)

10. Velocity field is defined by : $\vec{v} = y\hat{\imath} - x\hat{\jmath}$
    Is the flow irrotational i.e. is the curl zero? (marks 2)
    Is the flow incompressible i.e. is the divengence zero? (marks 2)

11. Express surface integral $\int\int_S \vec{f}.\hat{n}dA$ using divergence theorem if $\vec{f} = e^x\hat{\imath} + e^y\hat{\jmath} + e^z\hat{k}$ and surface $S$ is defined by the surface of a cube defined by
    $|x| \leq 1; |y| \leq 1; |z| \leq 1$
    Only expression required in terms of "e". Do not evaluate using calculator. (marks 4)

12. In the figure given below thick line shows a curve and the dotted ones are its projection on surfaces (for additional clarity)




(a) Which one is the osculating plane? (marks 1)

(b) Which of the vector a, b, c shows the tangent vector direction? (marks 1)

(c) of the vector a, b, c which is the principal normal vector? (marks 1)

(d) Which plane is normal plane? (marks 1)