

## Engineering Mathematics for Advanced Studies

**Vector Calculus: Assignment 02**Deadline – 12:00pm on Wednesday 23<sup>rd</sup> Oct. 2019

- 1) Consider motion of the satellite in an elliptical orbit given by varying position vector  $a\cos(t)\hat{i} + b\sin(t)\hat{j}$ . Find velocity and acceleration. (2 marks)
- 2) Certain force field is given by following equation:  $f = x^2\hat{i} + y^2\hat{k}$ . What is the direction along which (3 marks)
- A) Movement will be most difficult?
- B) Movement will happen by due to force field?
- C) Direction in which the movement will result in net work done by the force field to be zero.
- 3) Find unit normal or unit surface normal to the following curves and the surface at the given point:  
 a)  $ax + by + cz + d = 0$  at  $P(x_p, y_p, z_p)$       b)  $x^2 + y^2 + 2z^2 = 26$  at  $P(2, 2, 3)$  (2 marks)
- 4) Find Directional derivative  $D_v f$  of  $f = xyz$  at  $P(-1, 1, 3)$  along vector  $v = \hat{i} - 2\hat{j} + 2\hat{k}$  (2 marks)
- 5) Consider flow field given by velocity vector  $v = y\hat{i}$  Show that this flow is incompressible. ( Hint: Evaluate divergence of the flow field) (1 marks)
- 6) Find Laplacian of the following field  $f = e^{2x}\sin(2y)$  (2 marks)
- 7) For the flow field  $v = x\hat{i} + y\hat{j} - z\hat{k}$ , is the flow incompressible? Is the flow irrotational? (2 marks)
- 8) Evaluate line integral  $F(r) = 5z\hat{i} + xy\hat{j} + x^2z\hat{k}$  along (4 marks)
- a) Straight line joining point A(0,0,3) and B(0,0,5)
- b) a semicircle of radius 1 in the plane xz (positive) and with center at C(0,0,4) (hint: refer Example 3: Dependence of a line integral on path, pg 467 Kreyszig ed 8<sup>th</sup>)
- 9) Evaluate the surface integral  $\iint_S F \cdot n dA$  for  $F = [x^3, y^3, z^3]$  for spherical surface  $S = x^2 + y^2 + z^2 = 9$  ( Hint: Use divergence theorem) (2 marks)