
Final Exam - Module Probability and Statistics
Engineering Mathematics for Advanced Studies
 IIT Dharwad
 Autumn 2019

Date - 22nd Nov. 2019
 Total time - 1 Hour 45 minutes (06:00pm - 07:45pm)
 Maximum score - 60
 Rule for absentee - Minimum 30% penalty, discuss reasons absense in person to get a chance for re-test.
 Note:

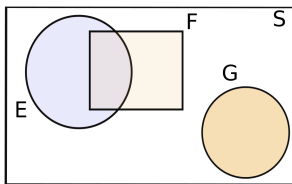
1. Combination is represented as $\binom{n}{r}$ or nC_r for selecting r out of n
2. Permutation can be represented as nP_r for arranging r out of n
3. $P(A)$ expresses the probability of event A
4. A^C implies compliment of A ; $E(*)$ implies expected value for the random variable or its function *
5. Do not evaluate exact value unless stated explicitly. Provide the simplified formula or expression in terms of the variables. e.g.
 $\frac{\binom{10}{2}\binom{8}{3}}{\binom{10}{5}}$ or $\frac{n-k}{(n-r)!}$

SESSION A - This is CLOSED BOOK session of the exam

Time - 25 minutes (06:00pm-06:25pm)

1. Which *all* of the following is true for the Venn diagram shown here:

(Marks 4)



- (a) $P(E \cup G) = P(E) + P(G)$
 - (b) $P(E \cup G) = P(E) + P(G) - P(E \cap G)$
 - (c) $P(E^C \cap F) = P(F^C) - P(E)$
 - (d) $P(E^C \cap F) = 1 - (P(F^C) - P(E))$
2. A state traffic department is contemplating of number plate schema where for each car the number plate will be of the format "AB pq XXXX". 'AB' is a fixed set of alphabet in fixed sequence chosen from 26 capital letters A-Z assigned to the state; 'pq' indicates 2 digits - each of which can range from 0 to 9 but both can not be 0. XXXX is another set of 4 numbers each of which can range from 0-9, however all can not be zero at the same time. Four numbers need not to be same always even though the symbol used contains repetition.
 - (a) Which of the following answers correctily mentions maximum number of possibilities (if your answer is not listed, please choose option "other" and write the answer & a brief explanation how you arrived at the answer

(Marks 2)

- i. $(10^4 - 1)(10^2 - 1)$
- ii. $(26)(25)(10P_2)(10P_4) - 2$
- iii. $\frac{(10P_2)(10P_4)}{2!4!}$
- iv. $(10P_2)(10P_4) - 4$
- v. Other : _____

- (b) If the sequence of the two numbers 'pq' is to be interpreted as a single number (e.g. p=2 and q=5 reads out the number 25) which can range from 1 to 45 what will be the total number of possibilities (Marks 1)

ANSWER: _____

3. If a random variable Y is defined as the number of times the head turns up in 3 trials of a fair coin being tossed which of the following is the correct probability of Y=3? (Marks 2)
- (a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

ANSWER: _____

4. Which of the following is true (Encircle the correct option): (Marks 2)
- (a) Set of the events which obey Poisson's distribution is a subset of the set of events that follow Binomial distribution
- (b) Set of the events which obey Binomial distribution is a subset of the set of events that follow Poisson distribution

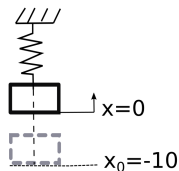
5. For a sample space S encompassing three events E, F, and G, if it is given that $P(E) = 0.3, P(F) = 0.8, P(G) = 0.2$, which of the following can NOT be true? (Marks 2)
- (a) Events E and F are mutually exclusive events
- (b) Events F and G are mutually exclusive events
- (c) Events E and F are independent events
- (d) Events F and G are independent events

ANSWER: _____

6. Conditional probability $P(E|F)$ _____ Probability that both event E and F happen. (Marks 2)
- (a) $>$ (b) $<$ (c) \leq (d) \geq

ANSWER: _____

7. Which of the following correctly describes sample space of the outcome where experiment involves measuring the vertical deflection of a spring-mass-damper system with initial displacement of 10 units and initial velocity of 0 units per second? (Marks 2)



- (a) $X = \{x : -10 \leq x \leq 10\}$
- (b) $X = \{x = -10, -9, -8, \dots, -1, 0, 1, \dots, 9, 10\}$
- (c) $X = \{x = i, \quad i = 1, 2, 3, \dots, 19, 20\}$
- (d) $X = \{x : |x| \leq 5\}$

ANSWER: _____

8. Which one of the following is correct? (Marks 2)
- (a) $E(g(x)) = \sum_{i=1}^{i=n} x_i \cdot p(x_i) \cdot g(x_i)$
- (b) $E(g(x)) = \sum_{i=1}^{i=n} p(x_i) \cdot (g(x_i))^2$
- (c) $Variance(X) = E((X - \mu)^2) = E(X^2) - \mu^2$ where $\mu = E(X)$
- (d) $E(X^2) = \sum_{i=0}^{i=n} (p(x_i) \cdot \frac{x_i^3}{3})$

9. True or False - Variance of a variable X with probability p(x) is analogous to the concept of center of gravity of any rod whose variable density is p(x) locates coordinate x. (Marks 1)

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Date - 22nd Nov. 2019

Maximum score - 40

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SESSION B- This is OPEN BOOK session of the exam

Time - 80 minutes (06:25pm-07:45pm)

1. For some experiment to be performed by a researcher there are two observations - event A that a specific component fails during the 10 hours of operation and event B that the amplitude of the vibrations crosses certain threshold at least once in the 10 hours.

	PASS	FAIL	
Threshold amplitude crossed	4500	2000	6500
Did not cross threshold amplitude	3000	500	3500
	7500	2500	

- (a) What is the conditional probability that component will fail if high amplitude vibrations are observed? (Marks 2)
- (b) What is the conditional probability that high amplitude vibrations were observed given that the component failed? (Marks 2)
- (c) What is the probability that any randomly selected component will fail? (Marks 2)
- (d) What is the probability that any randomly selected component will result in amplitude that will cross the threshold value during the test? (Marks 2)
2. The budget of 20 crores is to be split across 5 start-up funds applying for the funding to the funding agency. The budget can be only allotted in multiple of crore. In how many different ways split cn happen if
- (a) Each start-up should at least get one crore funding? (Marks 2)
- (b) It is allowed to completely reject funding to one or multiple start-ups? (Marks 2)
- (c) If each start-up is to receive minimum of 2 crores of the budget? (Marks 4)
3. A lab test has 95% accuracy in detecting a disease in a person with infection. However, it also has a false positive detection rate of 1% in the healthy people. i.e. it show positive result for 1 in 100 test cases which are actually not having any infection. 0.50% of the population are know to have the infection. What is the probability a person has the disease in reality given that lab test turns out to be positive for her?

(Hint: Take even D that the person has the disease and event E that the lab test is positive. We need to find

$$P(D|E) = P(DE)/P(E)$$

Use conditional probability to re-express the numerator and the Bayes formula to expand the denominator to introduce the event D related information from the the problem) (Marks 4)

4. A class consists of 5 MS students and 6 PhD students. It is required to rank them in the decreasing order of the marks they received in the final exam.
- (a) How many different rankings are possible? (Marks 2)

- (b) If the MS students are ranked amongst themselves and the PhD students are ranked amongst themselves, how many different ranking are possible? (Marks 2)
5. A class consists of 5 MS students and 6 PhD students.
- (a) A committee of 4 people involving 2 MS students and 2 PHD students is to be formed. There are 2 PHD students who share a TA responsibility in the same time slot when the committee is supposed to meet. Hence, both of them can not be on the committee, however, one of them can serve on the committee. How many different combinations are possible? (Marks 4)
- (b) In how many different ways can the selected committee of 4 people can be seated on a round table if the circular shift is also considered as a different seating? (Marks 2)
6. A father and daughter duo are playing some game which results either one of the player wining the game. Each time probability of father wining the game is ' $p = 0.7$ '.
- (a) However, being a "nice" father he wants to end the gaming session only when the daughter wins for the first time. What is the probability that they would end up playing for 30 minutes if each game takes around 2 minutes? (Marks 2)
- (b) Instead, if they decide to play for a fixed time of 30 minutes, what is the probability that the daughter will win 10 times? (Marks 4)
7. A civil construction contractor gives floor construction tiles free to his customer. Tiles cost him Rs. 500 per sq. feet. Customer get their rooms built by this contractor and pay the contractor for the labor and machinery as usual. His customer come with a pre-decided choice for getting constructed square rooms with the options for side length being one of the 10 ft, 20ft, 30ft, 40ft. Customer has probability of 0.2, 0.4, 0.3, and 0.1 respectively of choosing those room sizes. What is the budget to be earmarked for purchasing the tiles per customer who approaches the contractor? (Marks 4)