PDE-Assignment #2

Engineering Mathematics for Advanced Studies

IIT Dharwad

Autumn 2019

Submission - Thursday 7th Nov. 2019 10:00am (in-class submission)

Late penalty - 0-24 hrs late* 30%, 100% for more than 24 hrs (penalty starts from 10:01 am, 7th Nov. 2019 ☺)

Total marks - 10

1. For equation $\nabla^2 u = 0$:
   (a) What is the standard name? (1 Mark)
   (b) What do the independent variables (for which u is defined as dependent variable) "typically" represent? (1 Mark)
   (c) Please provide the explicit equation for $u = u(R, \theta, \phi)$ where R is radial distance from a point and θ and ϕ are two angles forming *spherical* coordinate system (1 Mark)
   (d) Please provide the explicit equation for $u = u(R, \theta, z)$ where R is radial distance from a point, z is the axial distance, and θ is angle forming *cyllindrical* coordinate system (1 Mark)
   (e) Please provide the explicit equation for $u = u(t, v)$ where (t, v) for a *cartesian* coordinate system (1 Mark)
   (f) Please provide the explicit equation for $u = u(R, \theta)$ where R is radial distance from a point and θ is angle forming *polar* coordinate system (1 Mark)

2. Is $\nabla^2 u$ operation invariant under following coordinate transformation from $x - y$ coordinates to $x^* - y^*$ coordinates:
   (a) Translation - defined by $x^* = x + a$ and $y^* = y + b$ where a and b are constants (2 Mark)
   (b) $\left\{ \begin{array}{c} x^* \\ y^* \end{array} \right\} = \left[\begin{array}{cc} cos(\alpha) & -sin(\alpha) \\ sin(\alpha) & cos(\alpha) \end{array} \right] \left\{ \begin{array}{c} x \\ y \end{array} \right\}$ where α is a constant (2 Mark)