ODE-Assignment #2

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Thursday 10th Oct. 2019 5:30pm
Total Score -10 marks
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 10th Oct. 2019!)

1. Give standard form for : i)Bernoulli eqn. ii) Logistic eqn. iii) Riccati eqn. (We did not cover Riccati eqn. in class; you can locate it in Kreszig using keyword list at the end of book). What is the difference in Bernoulli eqn. and Logistic eqn? Identify these 3 equations if those are Linear or Non-linear ODEs? (marks 1)

2. Given an Riccati equation $y' = x^3(y-x)^2 + \frac{y}{x}$ reduce it to Bernoulli equation by substitution $w = y - x$ and solve. (marks 2)

3. Take a separable ODE form: $\frac{dy}{dt} = \frac{g(t)}{f(y)}$ Verify if it does satisfy the exactness condition (marks 1)

4. We will explore response of a Spring-Mass system to:

   (a) Free Oscillations: (Hint: Note the experience that it is an oscillatory motion and use the usual suspects as guess solution!) (marks 1)

   $$my'' + ky = 0; \qquad y(0) = a; \qquad y'(0) = 0$$

   Will the amplitude shoot to infinity? (Yes/No)

   (b) Can you draw amplitude y_{max} vs ω plots for the above system with oscillatory forcing function on the RHS instead of zero. i.e. $my'' + ky = F_0 cos(\omega t)$ (marks 1)

5. For $y = y(t)$, consider equation $y" - y = 0$. This is a second order linear homogeneous autonomous equation. Take a pause and verify all the adjectives used earlier.

   (a) Can $y = e^{-t}$ be a solution?

   (b) Can $y = e^{t}$ be a solution?

   (c) Can $y = 4e^{t} - 3e^{-t}$ be a solution?

   (d) Observe that (c) is combination of (a) and (b). Can you please mention which of the adjective(s) used above is relevant in this scenario? (marks 1)

6. Solve:

   (a) $4y'' - 4y' - 3y = 0 \qquad y'(-2) = -\frac{e}{2} \qquad y(-2) = e$ (marks 1)

   (b) $y'' + 0.2y' + 4.01y = 0 \qquad y'(0) = 2 \qquad y(0) = 0$ (marks 1)

      i. Solve above ODE
      ii. Plot/Sketch the solution
      iii. What is the significance of each constant associated in the equation above? (You may want to play with the number to see how the plot changes in case you are comforable with tools like MATLAB)

7. Reading assignments - Just Answer "Yes" if you have read these sections, else answer "No": (Honor system) (marks 1)

   (a) Read Section 2.5 Modelling: Free Oscillations in Kreyszig

   (b) Read Section 3.3 (Homogeneous System ...) and section 3.4 (Criteria for Critical Points Stability) in Kreyszig