ODE-Assignment #1 (revised)

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Monday 23rd Sept. 2019 5:30pm
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 23rd Sept. 2019!)

1. Specify Order/Degree/Linear-NonLinear/Homogeneous-Nonhomogeneous parameters for the following set of equations for y(x)? (marks 1)

No.	Equation	Order (N)	Degree (N)	Linear(L) or Non-linear (NL) (L/NL)	Homogeneous(H) / Non-homogeneous(NH) (H/NH)
1	$y''' = e^x$				
2	$y'' + 2y' + 2y = 0$				
3	$x + yy' = 0$				
4	$y'/x + x = 0$				
5	$y' + sin(y) = x$				
6	$(y'')^3 + log(y) = x$				
7	$y'' + 3yy' + y = cos(t)$				
8	$y'' + (y')^2 + 4y = 2x$				
9	$y'' + (y')^2 + 4y = e^x$				
10	$y'' + (y')^2 + 4y = 0$				

2. Please give/formulate/conceptualize two practical day to day life non-academic examples which can be represented by an ODE.

(marks 1)

3. Please read the direction fields (Kreyszig section 1.2 "Geometrical meaning of y'=f(x,y) direction fields" in Kreyszig; it was not covered in detail in the class). Sketch by hand the direction fields for the following: (marks 1)

   (a) $y' = x^2$
   (b) $y' = y^2$
   (c) $y' = x + y$
   (d) $y' = \frac{-x}{y}$

4. An experimentalist observes that light absorption in a very thin transparent layer is proportional to the thickness of the layer and to the amount of the light incedent on that layer. Please formulate this in terms of the differential equations. Please clearly mention the symbols/variables used and their meaning in this context.

(marks 1)

5. Solve following differential equation (marks 1)

   (a) solve

$$y' = -\frac{x^3 + 3xy^2}{3x^2y + y^3}$$

   (Hint: Test for exactness off ODE can help)

   (b) Is the final solution you stated up with above implicit or explicit solution (use internet to know what is implicit and explicit solution if required)

6. If temperature of a cake is 150 Degree C when it leaves oven and is 95 Degree C in 10 minutes later. Assuming only convection heat loss i.e. Newton's law of cooling lets explore how it reached practically to room temperature 21 Degree C. (marks 1)

   (a) State the explicit equation that gives temperature T at any given time t (expressed in minutes).
   (b) Provide Table of values t for which T(t) equals 47, 42, 32, 27, 24, 23, 22 respectively.

(c) Plot the relation T(t) along y axis as a function of time t along x-axis

7. Carefully read section 1.7 (Modelling of Electrical Circuits) in Kreyszig. Explore how the plots of I and Phase angle as function of time t change if (marks 1)

   (a) R becomes 2R in RL circuit

   (b) L becomes 2L in RL circuit

   (c) R becomes 2R in RC circuit

   (d) C becomes 2C in RC circuit
   (For each of above 4 sub questions superimpose two relavant plots before and after change; total 8 plots expected)
   Side comment - It is suggested that you have understood this text and the derivations presented in that. No marks for reproducing any of that here)

   (e) If isothermal location are given by family of curves $2x^2+y^2 = constant$, what are the orthogonal trajectories along which heat flow is expected (assuming no heat source or sync in the vicinity and homogeneous material) (Hint: Reading Example 1 in section 1.8 Orthogonal Trajectories of curves may help)

8. We will explore $e^{(a+b)t} = e^{at}e^{bt}$ in this example. Solve $y' = (a+b)y$ by (marks 0 !)

   (a) Any method of you choice

   (b) using product rule for differentiation and assuming $e^{at}e^{bt}$ as a possible solution.

   (c) Can we say $e^{(A+B)t} = e^{At}e^{Bt}$ where A and B are matrices following proof in (b) above extended to $y = \{y_1, y_2, y_3, \}$....? vector. If not, why?

9. Solve (marks 1)

   (a) $y' = 5y + e^{-4t}$

   (b) $y' = 5y + e^{-5t}$

   (c) $y' = 5y + e^{5t}$

   (d) $y' = 3y + 5e^{2it}$

   (e) $y' = 2y + 3cos(t) + 4sin(t)$

10. Express sin(5t) + cos(5t) as $Rcos(\omega t - \phi)$ (marks 1)
    R =? ϕ =?

11. If principal p is deposited at time t=0, with no further addition, with an interest rate a% per year. (marks 1)

    (a) What is the ODE to model rate of growth of the investment

    (b) Solutions to get worth of investment after any given time t years

    (c) Look up "Rule of 72" online and observe if you answer in (b) is in line with that or not.

    (d) If a was specified as interest rate per day instead of per year and t in days instead of years, what would (b) become? Should you get more or less worth of investment comparative to interest a defined per year for the same duration i.e. say 10 years or 3650 days