In-Class Test #1 - Module Numerical Methods

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Time - 20 minutes
Maximum score - 20
Rule for absentee - Minimum 30% penalty, discuss reasons absense in person to get a chance for re-test.
Date - 14th Nov. 2019

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1. Free run out condition on the cubic spline means: (mark 2)

   (a) Slopes on both ends are zero

   (b) ~~Curvature~~ *Second derivative of curve* at both ends are zero

   (c) ~~Curvature~~ *Second derivative of curve* at both end nodes equals the respective values at neighboring node

   (d) Slope at both end nodes equals the respective values at neighboring node

   ANSWER: ___(b)______________________________

2. A student ABC uses following criteria to stop numerical iterations:
   $error = (x_{new} - x_{prev})$
   if $error \leqslant tolerance$
   exit

   Another student XYZ uses
   $error = (\frac{x_{new}-x_{prev}}{x_{new}})$
   if $error \leqslant tolerance$
   exit

   Which one is more appropriate approach to ensure universatality of the subroutine across different applications - ABC's or XYZ's? (mark 1)

   XYZ approach is more professional as it uses relative error instead of absolute error. Note, however, both have common flaw - absolute value of the error $||error||$ should have been used in if statement

   Which practically precaution is necessary for XYZ appropriate approach? (mark 2)

   Precaution necessary in case of XYZ is that it involved division by a x_{new}. Hence it is imperative to ensure that $x_{new} \neq 0$ i.e. additional if statement :

   if $x_{new} \neq 0$
   error=$\|\frac{x_{new}-x_{prev}}{x_{new}}\|$

3. True or False

   (a) Global error of the Trapezoidal method is of the smaller order than the order of local error of the same method (mark 1)

   __________TRUE_________Trapezoidal methods - Global error $\mathcal{O}(h^2)$ and Local error $\mathcal{O}(h^3)$

   (b) Simpson's rule belongs to Newton-Cotes category however Gauss quadrature is not Newton-Cote's formula (mark 1)

   __________TRUE____

(c) Langrange interpolation is a polynomial interpolation (mark 1)

__________TRUE____

(d) Numerical finite difference scheme is unconditionally stable for Elliptical PDE (mark 1)

__________TRUE____

(e) Numerical finite difference scheme is unconditionally stable for Parabolic PDE (mark 1)

__________FALSE__________For parabolic equation there is conditional stability. (See section 19.6, $r = \frac{k}{h^2} \leq \frac{1}{2}$)

(f) Higher order differential equation can be solved using appropriate choice of Euler method (mark 1)

__________TRUE____________Typical process: Convert the equation into a system of first of linear differential equations and use Euler method in higher dimensions.

(g) Newton-Raphson methods needs more evaluations of functions compared to Newton-Secant method (mark 1)

__________TRUE___________Newton-Raphson evaluated both $f(x_i)$ and $f'(x_i)$ Newton-secant only evaluates $f(x_i)$each time (and readily gets $f(x_{i-1})$ from previous calculation).

4. If instructor gets equally spaced 11 points from equation $y = 2x^2 - 4x + 2$ and poses the numerical integration problem to students. Student ABC integrates using the Simpson's Rule while the XYZ uses Trapezoidal rule (Choose all correct answers) (mark 2)

   (a) ABC's answer will match analytical value of the integration (ignore round-off errors)

   (b) XYZ's answer will match analytical value of the integration (ignore round-off errors)

   (c) Both ABC and XYZ will have some error, but ABC's error would be smaller that XYZ's error

   (d) Both ABC and XYZ will have some error, but ABC's error would be larger that XYZ's error

   (e) If one uses midpoint rule, accuracy will be better that both ABC and XYZ

   Note that the given function is quadratic in nature and hence Simpson's rule which uses parabolic approximation would correctly estimate integral.

5. For solving system of linear equations $Ax = b$, given the inverse A^{-1} is readily available, state how many operations would be required to get x for a given b ? (mark 2)
   n^2 where $A_{n\times n}$

6. Given that variable x can be have numerical error bounded by β_x and y can be have numerical error bounded by β_y , what will be the absolute error bound for $z = x + y$ (mark 2)
   err=$\beta_x + \beta_y$

7. If one reduces the step size by half, the numerical scheme which has error ~~of the order 3~~ $\mathcal{O}(h^3)$, would have *how much improvement/reduction* ~~change~~ in its accuracy ? (marks 2)
   $h_{new=}\frac{1}{2}h_{old} \implies error\ \mathcal{O}(h^3_{new})$ i.e. $\mathcal{O}(\frac{h^3_{old}}{8})$ Accuracy will improve as truncation error will become smaller by a factor of 8.