Final Exam - Module Numerical Methods

Engineering Mathematics for Advanced Studies

IIT Dharwad

Autumn 2019

Date - 20th Nov. 2019

Total time - 2 Hours (8:30am-10:30am)

Maximum score - 25

Rule for absentee - Minimum 30% penalty, discuss reasons absense in person to get a chance for re-test.

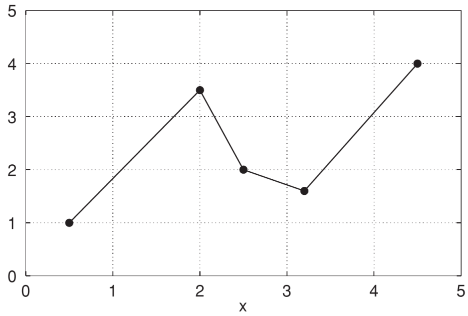
SESSION A - This is CLOSED BOOK session of the exam

Time - 30 minutes (8:30am-9:00am)

1. Very large values in the diagonal of the matrix A^{-1} compared to the rest of the values suggest ill-conditioning (TRUE or FALSE) (mark 2)

   ANSWER: ________________________________

2. Which of the following is the most appropriate possible description for the y-axis in the following plot if $g(x)$ represents a cubic spline (mark 2)



   (a) $g"(x)$
   (b) $g(x)$
   (c) $\int g(x)$
   (d) $g'(x)$

3. The following expression $f(x)$ describes (Marks 2)

$$f(x) = \sum_{j=0}^{n} y_j \prod_{i=0, i\neq j}^{n} \frac{x - x_i}{x_j - x_i}$$

   (a) Cubic spline
   (b) Parabolic Spline
   (c) Lagrange interpolation
   (d) Piecewise linear interpolation

4. Please match a,b,c and p,q,r correctly: e.g. $a \mapsto p$, $b \mapsto q$ $c \mapsto r$ (Marks 3)

Error	**Name**
a) $y(x+h) \approx y(x) + hy'(x)$	p) Round off error
b) $1.2535 \approx 1.25347656$	q) Programing error
c) errorValue<Tolerance instead of $\|\|errorValue\|\| < Tolerance$	r) Truncation error

5. If instructor gets equally spaced 11 points from equation $y = 2x^2 - 4x + 2$ and poses the numerical integration problem to students. Student ABC integrates using the Simpson's Rule while the XYZ uses Trapezoidal rule (Choose all correct answers) (mark 1)

(a) ABC's answer will match analytical value of the integration (ignore round-off errors)

(b) XYZ's answer will match analytical value of the integration (ignore round-off errors)

(c) Both ABC and XYZ will have some error, but ABC's error would be smaller that XYZ's error

(d) Both ABC and XYZ will have some error, but ABC's error would be larger that XYZ's error

(e) If one uses midpoint rule, accuracy will be better that both ABC and XYZ

6. True or False

(a) For solving a parabolic PDE numerically Crank Nicolson scheme with spatial discretization of 0.4 units and time discretization of 0.1 is stable. (mark 1)

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(b) Midpoint rule is more accurate compared to the trapezoidal rule (mark 1)

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(c) Choleskey decomposition method for following matrix would give 9 times faster solutions of system of linear equation $Ax = b$ compared to that of normal Gaussian elimination approach (mark 2)

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 2 & 3 \\ 1 & 1 & 5 \end{bmatrix}$$

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7. Half band width of the following matrix is $w =$________ (mark 2)

$$A = \begin{bmatrix} 1 & 2 & 2 & 0 & 0 & 0 & 0 \\ 2 & 3 & 2 & 2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 2 & 2 & 0 & 0 \\ 0 & 1 & 2 & 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 2 & 3 & 2 & 2 \\ 0 & 0 & 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{bmatrix}$$

8. Please name the following method - (Marks 1)

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

where

$$y^*_{n+1} = y_n + hf(x_{n,}y_n)$$

and $y' = f(x, y)$ with (x_0, y_0) is given.
Options: a) Euler b) Modified Euler (Heun method) c) Runge-Kutta d) Crank-Nicolson
ANSWER : _____________________________________

9. For $c = a + b$, what is error bound of c if a and b are known to have error bound of 0.000005 each: _________________ (Marks 1)

10. Consider the interval $[a, b]$ for integration of some function $f(x)$. Lets say we divide it into subintervals $[x_i, x_{i+1}]$ which is considered as two panels with 3 nodes $x_i, x_i + \frac{h}{2}, x_i + h$. Following expression gives formula for Simpson's rule of integration for the discretization with this step size h.

$$S_i = \frac{h}{6}\left[f(x_i) + 4f\left(x_i + \frac{h}{2}\right) + f\left(x_i + h\right)\right]$$

(a) The same interval $[x_i, x_{i+1}]$, if we divide into four panels and repeat the Simpson's rule to approximate integral with first two panels once and add it to that of the 3rd and 4th panels. (i.e. apply Simpson's rule for same inteval twice with halved step size) can you write the expression for this refined estimate $S_i^{(2)}$ in terms of h and the values of function at 5 nodes. (Marks 4)

(b) By subtracting S_i from $S_i^{(2)}$ can you get the expression of the error that was corrected by halving the step-size in terms of the values of the function f at 5 nodes (Marks 2)

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SESSION B- This is OPEN BOOK session of the exam
Time - 90 minutes (9:00am-10:30am)

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11. On a certain processor, diagonal matrix $C_{1000\times 1000}$ matrix equation took 50 ms to complete Gaussian elimination using Gaussian elimination approach.We focus only on the row alimination process which forms major chunk of the total computational load.

    (a) If another matrix D is given as below, roughly how much time (in micro seconds) is expected (ball-park estimate) for its row eliminations process using algorithm which optimizes special characteristics of the matrix D? (mark 4)

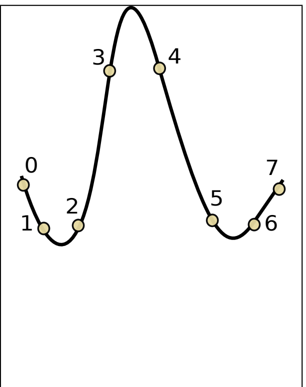
    (b) How much time is estimated for the matrix E given by $E = \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}$ (mark 4)

$$C = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 2 & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{bmatrix}_{1000\times 1000} \qquad A = \begin{bmatrix} 1 & 2 & 2 & 0 & 0 & 0 & 0 \\ 2 & 3 & 2 & 2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 2 & 2 & 0 & 0 \\ 0 & 1 & 2 & 1 & 2 & 2 & 0 \\ 0 & 0 & 1 & 2 & 3 & 2 & 2 \\ 0 & 0 & 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{bmatrix};$$

$$B = \begin{bmatrix} 1 & 4 & 0 & 0 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \end{bmatrix}; \quad D = \begin{bmatrix} A & A & B & 0 & \ddots & 0 & 0 \\ A & A & \ddots & B & 0 & \ddots & 0 \\ B & \ddots & A & \ddots & B & 0 & \ddots \\ 0 & B & \ddots & \ddots & \ddots & B & 0 \\ \ddots & 0 & B & \ddots & A & \ddots & B \\ 0 & \ddots & 0 & B & \ddots & A & A \\ 0 & 0 & \ddots & 0 & B & A & A \end{bmatrix}_{1001\times 1001}$$

12. Given velocity data:

No.	Time	Velocity
0	0.0	0.12507
1	2.0	-0.12828
2	4.0	-0.10022
3	6.0	0.66500
4	8.0	0.66500
5	10.0	-0.10022
6	12.0	-0.12828
7	14.0	0.12507



Please set-up the matrix to arrive at cubic spline for this data with parabolic run out condition.

$$\begin{bmatrix} a & b & c & 0 & 0 & 0 & 0 \\ \frac{h}{6} & \frac{2h}{3} & \frac{h}{6} & 0 & 0 & 0 & 0 \\ & & & & & & 0 \\ & & & \ddots & & & \\ 0 & & & & & & \\ 0 & 0 & & & & & \frac{h}{6} \\ 0 & 0 & 0 & 0 & p & q & r \end{bmatrix} \begin{Bmatrix} g"(x_0) \\ g"(x_1) \\ \\ \vdots \\ \\ g"(x_6) \\ g"(x_7) \end{Bmatrix} = \begin{Bmatrix} H_1 \\ \frac{f(x_2)-2f(x_1)+f(x_0)}{h} \\ \\ \vdots \\ \\ \\ H_2 \end{Bmatrix}$$

(a) What are the values of a, b, c, p, q, r and H_1 and H_2 in above set-up worked out for your reference? (marks 4)

(b) Considering only the points #4,5,6,7 in above data, please explicitely write Langrange polynomial. (marks 4)

13. Using Newton-Raphson method, please find at what time will the two processes whose temperatures T_1 and T_2 governed by $T_1(t) = 100\cdot(1-e^{(-0.2t)})$ and $T_2(t) = 40\cdot e^{-0.01t}$ will reach equal temperature? Take initial guess $t_0 = 1$. Find t_1, t_2, and t_3. (Report numbers till 4th decimal throughout the answer) (Marks 6)

14. If student A uses forward difference method by dividing the span of 100 m in increaments of 1cm and other student B uses 5cm step size for same problem but uses central difference method. Whose accuracy will be better? or both will have same accuracy in estimating slope? (Marks 2)

15. Very large values in the main diagonal of the matrix compared to other values in the matrix suggest a well conditioned matrix. TRUE or FALSE (Marks 2)

16. If the side of a rectangle can be measured with a relative accuracy 0.0005, what is the relative error expected in the area of the rectangle? (Marks 2)

17. Express following differential equations as a system of first order differential equations (Marks 3)

$$\frac{d^4y}{dx^4} - 2\frac{d^2y}{dx^2} + 3y = 0$$

18. Demonstrate Gauss-Siedel method upto 3 iterations for the following system of linear equations with initial guess $[1, 1, 1]$: (Marks 4)

$$5x_1 + x_2 + 2x_3 = 19$$

$$x_1 + 4x_2 - 2x_3 = -2$$

$$2x_1 + 3x_2 + 8x_3 = 39$$