In-Class Test #1
Engineering Mathematics for Advanced Studies
(Module - Linear Algebra)
IIT Dharwad
Autumn 2019

Time - 30 minutes
Maximum score - 20
Rule for absentee - Minimum 30% penalty, discuss reasons absense in person to get a chance for re-test.

Worked out solutions on suppliments are must for some problems.
Please ensure to write Question number in a box as a heading to the upcoming answer on suppliments e.g.
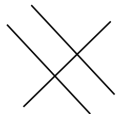
Question 1

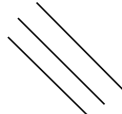1. Observe following three equations and choose best option amongst following

$$x + y + z = 1$$
$$x + y + z = 3$$
$$x - y = 1$$

A) Singular   B) Singular   C) Singular   D) Non-singular

   Answer: _ _ _ _ _ _ _ (marks 2)

2. Mr. Doshi receives Rs. 135000 as a bonus payment. He splits it in 3 divisions and invests in shares of 3 companies A, B, C respectively. A year later he reviews that investment to find that invested amount in company A has become 3 times the original and that in the company B became 2 times and the company C shares remain unchanged. Total at the end of year was Rs. 200000. It is given that initially he had invested in company A and B a total sum of Rs. 100000.
   (Attach detailed worked out solution on suppliment papers)

   (a) Represent this as linear system of equations $Ax = b$. Ensure that the equations are formed in the same sequence as the information is provided above i.e. last bit of information becomes the last equation in the set of equations. (1 mark)
   (b) Perform row elimination operations to get an upper triangular matrix U in $Ux = c$. Do the bare minimum operations required e.g. avoid fliping signs on both LHS and RHS of an equation. (1 mark)
   (c) State this set of row operations as a premultiplication matrix E which when multiplies A from left results in U. i.e. EA=U (3 marks)

3. Given that a matrix A in $Ax = b$ is a 5×7 matrix (row $\times$ column) with rank 3

(a) Here A is a linear transformation mapping from $\mathbb{R}^p$ to $\mathbb{R}^q$ What are values of p and q here?
Answer: $p =$ _ _ _ _ _ _ $q =$ _ _ _ _ _ _ (mark 1)

(b) What is the dimension of the **null** space
Answer: _ _ _ _ _ _ _ _ _ _ _ _ _(mark 1)

(c) What is the dimension of the **row** space
Answer: _ _ _ _ _ _ _ _ _ _ _ _ _(mark 1)

(d) What is the dimension of the **column** space
Answer: _ _ _ _ _ _ _ _ _ _ _ _ _(mark 1)

(e) What is the dimension of the **left null** space
Answer: _ _ _ _ _ _ _ _ _ _ _ _ _(mark 1)

(f) How many special solution should we expect
Answer: _ _ _ _ _ _ _ _ _ _ _ _ (mark 1)

4. Decide whether or not the following vectors are linearly independent?
(no marks awarded even for right answer if the required process/explanation is not provided. Attach worked out solution on suppliment papers.)

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

(a) Linearly independent?
Answer: _ _ _ _ _ _ _ _ (Yes/No)(marks 2)

(b) What is the dimension of the vector space spanned by those?
Answer: _ _ _ _ (marks 1)

5. Is following transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ a linear transformation ?
(no marks awarded even for right answer if the required process/explanation is not provided. Attach worked out solution on suppliment papers.)

$$T(x_1, x_2) = (x_2, x_1)$$

note- vector addition operations and multiplication by scalar is defined as:

$$\left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} + \left\{ \begin{array}{c} x_3 \\ x_4 \end{array} \right\} = \left\{ \begin{array}{c} x_1 + x_2 \\ x_3 + x_4 \end{array} \right\}$$

$$\alpha \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \left\{ \begin{array}{c} \alpha x_1 \\ \alpha x_2 \end{array} \right\}$$

Answer: _ _ _ _ _ _ _ _ (Yes/No) (marks 2)