Assignment #4

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Monday 9th Sept. 2019 5:30pm
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 9th Sept. 2019!)

1. Given that the internal corner edges of an aesthetically differentiating parallelopiped shaped water tank on the rooftop to be $(-1, -1, 2)$, $(1, 0, 2)$, $(0, 1, 2)$ with axis x$(1, 0, 0)$ and y$(0, 1, 0)$ parallel to the wall and the z axis$(0, 0, 1)$ along vertically upward directions and the $(0, 0, 0)$ being one of the corner vertex where structure is supported, can you estimate the water holding capacity of the same? (marks 1)

2. What is the determinant of the **inverse** of the following matrix:

$$\begin{bmatrix} 11 & 22 & 44 & 22 \\ 0 & 20 & 40 & 200 \\ 0 & 0 & 25 & 50 \\ 0 & 0 & 50 & 108 \end{bmatrix}$$

   (Please leverage at least 4 properties of determinants listed in the notes posted online to get the answer. No marks for explicit determinant calculations or just the final answer.)

   (marks 1)

3. Referring to the deteminant properties (and particularly section 4.3 "Formulas for Determinants" in Gilbert Strang, Lin Algebra and its applications) can you prove traditional formula for detereminant of a 3×3 matrix given below:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

   (marks 2)

4. Observe following transformation $A\boldsymbol{x} = \boldsymbol{u}$

   (marks 3)

$$\begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix} \boldsymbol{x} = \boldsymbol{u}$$

   (a) It is a mapping from $\mathbb{R}^p$ to $\mathbb{R}^q$. p= ——? q =———?

   For the following set of questions $\boldsymbol{x}$ is allowed only to be such that $\|\boldsymbol{x}\| = 1$.

(b) It is intended to maximize $\|A\boldsymbol{x}\|$. Can you think of a method to do so? (Hint: maximizing square of a quantity is equivalent to maximizing absolute value (magnitude) of a the quatity. Use the dot product concept for norm calculation and put to use Eigen vector knowledge. Feel free to use MATLAB or any other tool to get eigen values/vectors or quick matrix operations)

(c) Can the same procedure be repeated for finding minimum of $\|A\boldsymbol{x}\|$? What will be the difference compared to above maximization problem?

(d) Report the vector(s) in the input space that correspond to maxima and minima?

(e) Report the vector(s) in the output space that correspond to maxima and minima?

(f) Can you sketch the input space and the output space correspoding to $\boldsymbol{x}$ defined in (b)

5. Consider the following matrix A

$$A = \begin{bmatrix} 8 & 0 & 1 \\ 0 & 8 & 1 \\ 1 & 1 & 7 \end{bmatrix}$$

(marks 3)

(a) Give its characteristic equation. State the Eigen values.

(b) As per Cayley-Hamilton's theorem, every matrix satisfies its own characteristic equation. Can you write expression for the same?

(c) Now, can you express A as a spectral decomposition $A = Q\Lambda Q^T$. If not possible express in diagonal form $A = S\Lambda S^{-1}$

(d) Can you use (c) to find necessary powers of the A required in (b) ? Substitute in (b) and verify Cayley-Hamilton's theorem

(e) What you expect the value of the multiplication of the pivots of the A should be based on above answers?

(f) Is this positive definite or positive semi-definite or negative definite or negative semidefinite or indefinite matrix?

Note- Manual calculations of all matrix operations are not expected. Prefer using ready to use tools to get the necessary task done e.g. matrix multiplication, eigen value and vector computation etc.