Assignment #3

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Tuesday 27th August 2019 5:30pm
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 27th August 2019!)

For this assignment students are requested to read Section 3.4(Orthogonal Basis and Gram-Schmidt) in Gilbert Strang Linear Algebra and Its applications book, 4th Edition.

1. What is Orthonormal and Orthogonal basis? (1 mark)
2. Which 2 conditions qualify a matrix as Orthogonal matrix? Given an orthogonal matrix Q how to get its inverse leveraging the fact that it is an orthonormal matrix? (1 mark)
3. Gram-Schmidt process (informational review, no marks):
   (a) While defining a basis for vector space we need to ensure: i) basis vectors are independent and ii) those span entire vector space. Given a set of vectors in a vector space, we can arrive the basis defining the vector space spanned by those vectors using a simple intuitive way.
   (b) Begin one of the vectors (say v1) an take a normalized vector in that direction as first vector in the basis vectors (say q1)
   (c) Take the next vector (say v2) and subtract projection of that vector onto the first basis vector(q1) and subtract it from the original vector. This gives component of v2 that can not be expressed by e1. Again normalize it to get next basis vector(say q2).
   (d) For all other vectors repeat same procedure of subtracting their projections onto the basis vectors defined so far followed by normalization. e.g. in case of v3, subtract from it both the projections on q1 and q2 and normalize.
4. Using the Gram-Schmidt process given above, for following matrix, find the respective basis vectors that define the column space for A and B given below (1+1 marks)

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 3 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Compute determinant for A and B. Comment on your observation based on above findings. (2 marks)

5. For following matrix C find the orthonormal basis vectors that define column space. (1 mark)

$$C = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

   (a) State orthonormal matrix Q formed by q1, q2, q3 found above. Check if rows are orthonormal to each others (any one pair). Can you prove that for any orthogonal matrix Q rows are orthonormal? (1 mark)

   (b) Express each column as a combination of the previously found basis vectors. Express these 3 sets of equations as (wise) matrix multiplication (remember the column picture?). State if you observe any key pattern. (2 marks)

   (c) Isn't is obvious? Check the section on $A = QR$ decomposition in section 3.4 of reference book by Prof. Gilbert Strang.