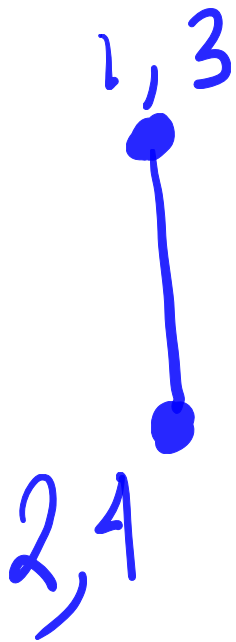


$$\begin{array}{ccc}
 G & \xrightarrow{f} & H \\
 \downarrow & & \downarrow \\
 V(G) & \longrightarrow & V(H) \text{ s.t. } uv \in E(G) \implies f(u)f(v) \in E(H)
 \end{array}$$

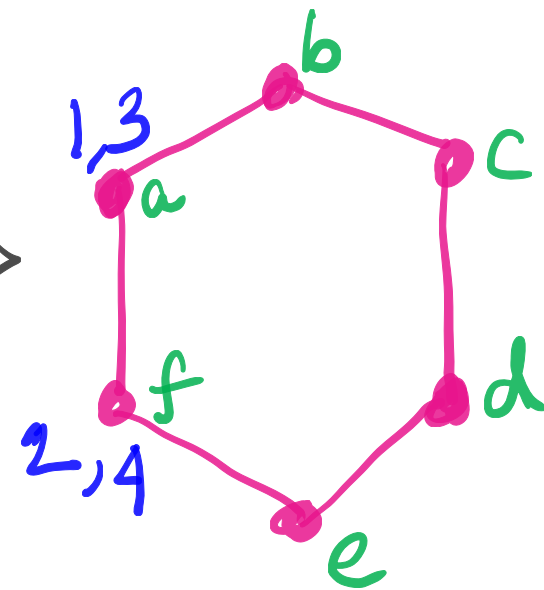
A bit on graph

homomorphisms.

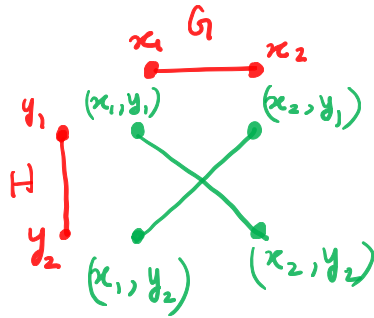
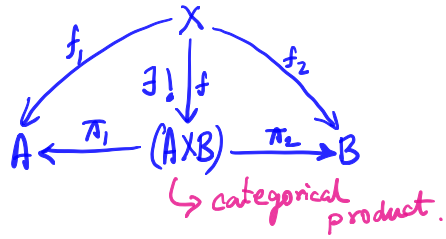


$$\phi(1) = \phi(3) = a$$

$$\phi(2) = \phi(4) = f$$



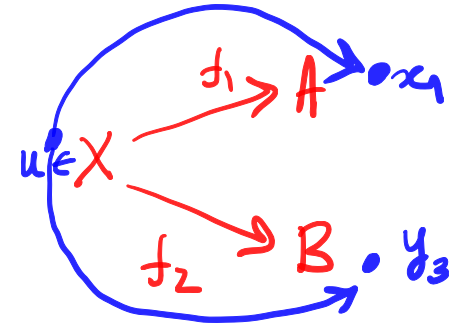
Products



$$\pi_1(u, v) = u$$

$$\pi_2(u, v) = v$$

$G \times H$

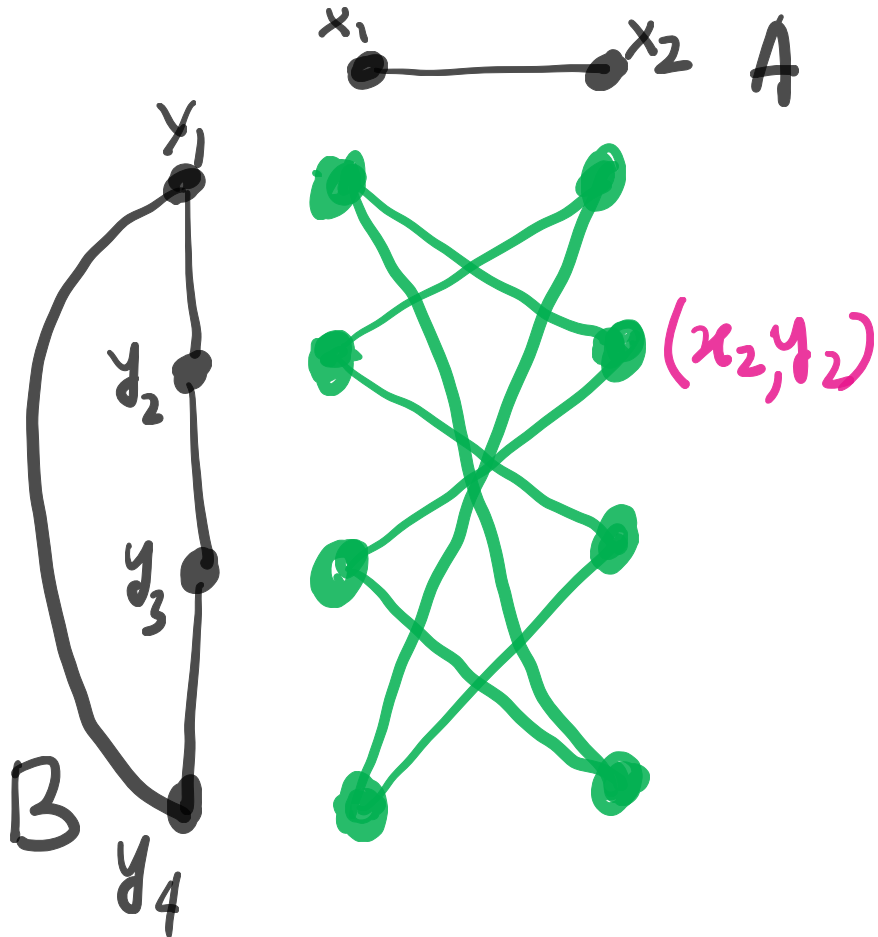


$$\begin{array}{ccc} X & \xrightarrow{f} & A \times B \\ \downarrow u & \mapsto & (f_1(u), f_2(u)) \\ & & (x_1, y_3) \end{array}$$

In graph category

— product exists.

It is $A \times B$.



Prove: $\chi(G \times H) \leq \min\{\chi(G), \chi(H)\}$.


Conjecture: $\chi(G \times H) \stackrel{??}{=} \min\{\chi(G), \chi(H)\}$
(Hedettenami's)

* Aditya + Shakil

$\Rightarrow$ False. Shitov (Annals of Mathematics).

Defⁿ K is multiplicative if
 $G \times H \rightarrow K \Rightarrow G \rightarrow K$ or $H \rightarrow K$

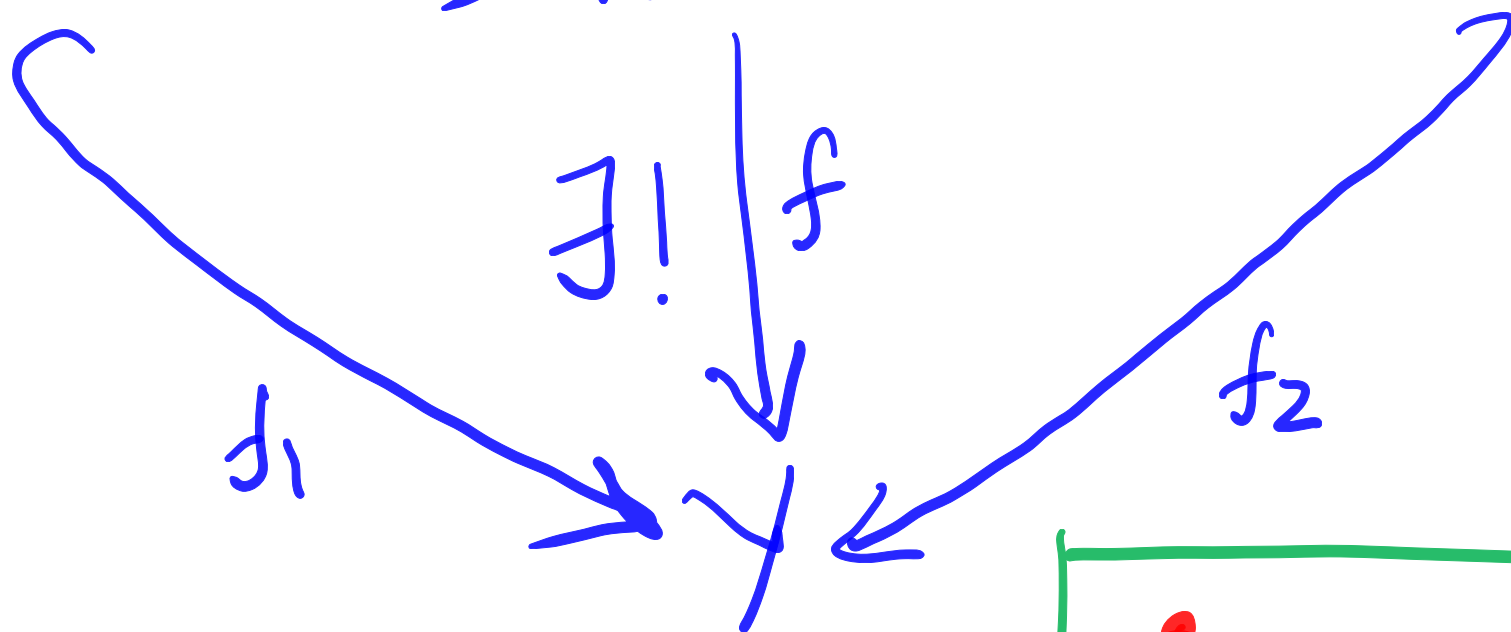
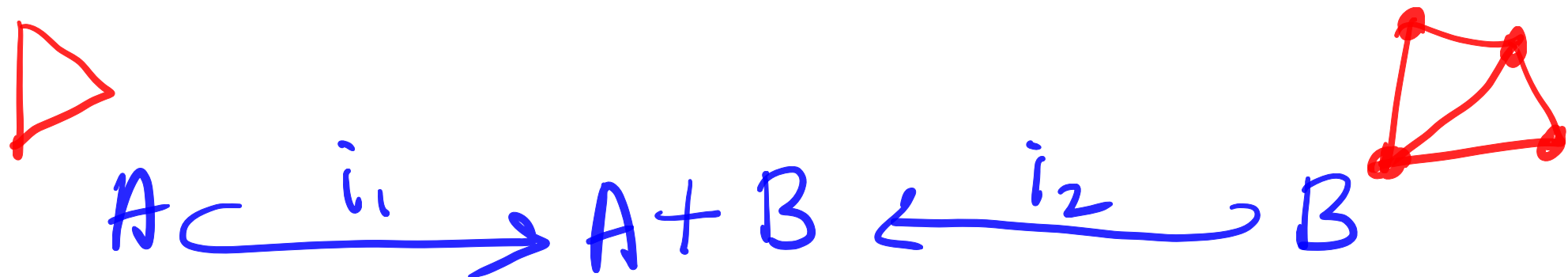
Hedetaniemi's conjecture (restated)

All complete graphs are multiplicative.

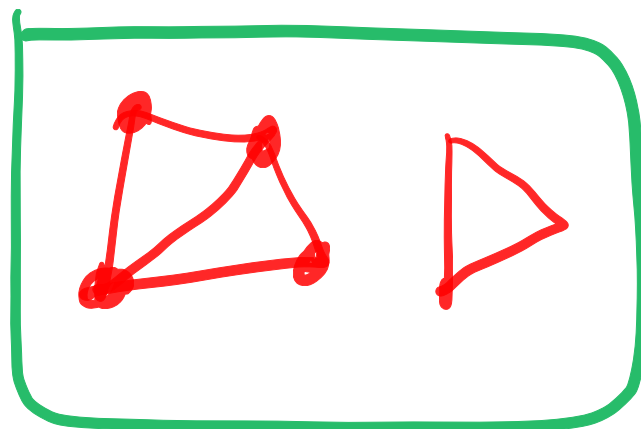
Advances: K_1, K_2, K_3 are multiplicative.

* Later we will introduce & discuss homomorphisms of colored mixed graph.

How about studying the analogous question there?



Coproduct



Properties:

isomorphism: bijective hom
whose inverse is
also hom.

$$(i) \quad G \times H \cong H \times G$$

$$(ii) \quad G \times (H \times K) \cong (G \times H) \times K$$

$$(iii) \quad G \times (H + K) \cong (G \times H) + G \times K$$

Exercises.

Homomorphism order. partial order.

$$\boxed{G \longrightarrow H} \quad \Longleftrightarrow \quad \boxed{G \leq H.}$$

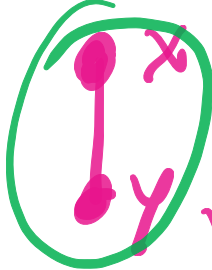
$$\boxed{G \longleftrightarrow H} \quad \Longleftrightarrow \quad \boxed{G = H.}$$

$$\boxed{G \not\longleftrightarrow H} \quad \Longleftrightarrow \quad \boxed{G < H.}$$

Density theorem: If $G \neq k_1$ or

$H \neq k_2$, then

if $G < H$, then $\exists X$ s.t. $G < X < H$.
✓

$G = a \rightarrow H =$ 

$\exists G \rightarrow H$

$H \not\rightarrow G$

$k_1 < k_2$

hom poset contains all other posets??

