Department of Mechanical Engineering
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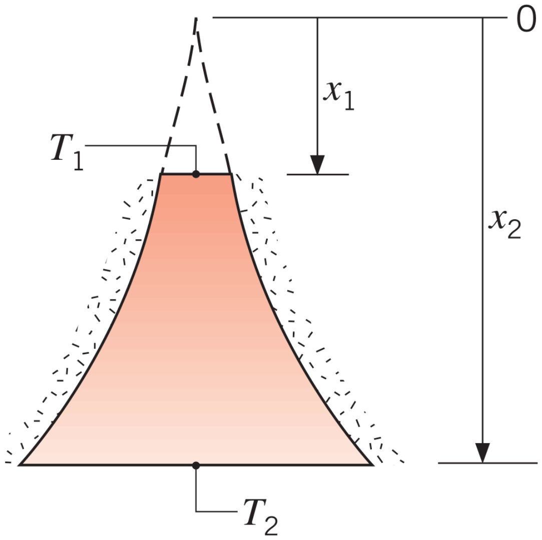
ME 301 – Heat Transfer

Problems

1. A truncated solid cone is of circular cross section, and its diameter is related to the axial coordinate by an expression of the form $D = ax^{3/2}$, where $a = 1.0\ \text{m}^{-1/2}$.



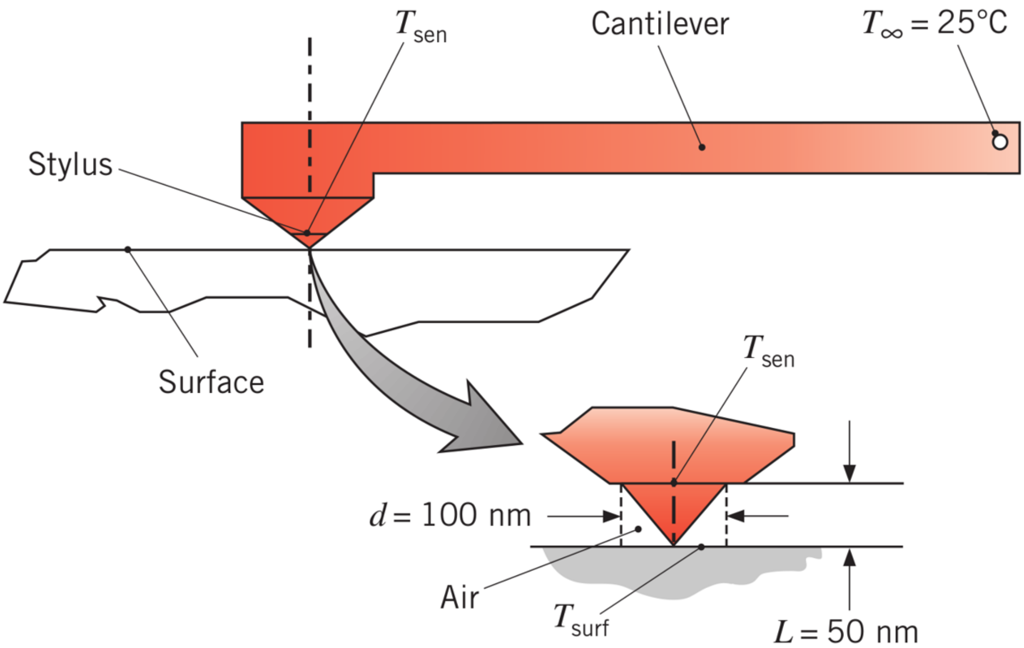
The sides are well insulated, while the top surface of the cone at x_1 is maintained at T_1 and the bottom surface at x_2 is maintained at T_2.

(a) Obtain an expression for the temperature distribution $T(x)$.

(b) What is the rate of heat transfer across the cone if it is constructed of pure aluminum with $x_1 = 0.075$ m, $T_1 = 100$ °C, $x_2 = 0.225$ m, and $T_2 = 20$ °C

2. A device used to measure the surface temperature of an object to within a spatial resolution of approximately 50 nm is shown in the schematic. It consists of an extremely sharp-tipped stylus and an extremely small cantilever that is scanned across the surface. The probe tip is of circular cross section and is fabricated of polycrystalline silicon dioxide. The ambient temperature is measured at the pivoted end of the cantilever as $T_\infty = 25\ °\text{C}$, and the device is equipped with a sensor to measure the temperature at the upper end of the sharp tip, T_{sen}. The thermal resistance between the sensing probe and the pivoted end is $R_t = 5 \times 10^6$ K/W.

Thermal conductivities of air and the probe tip material are 0.0263 W/m · K and 1.38 W/m · K respectively.

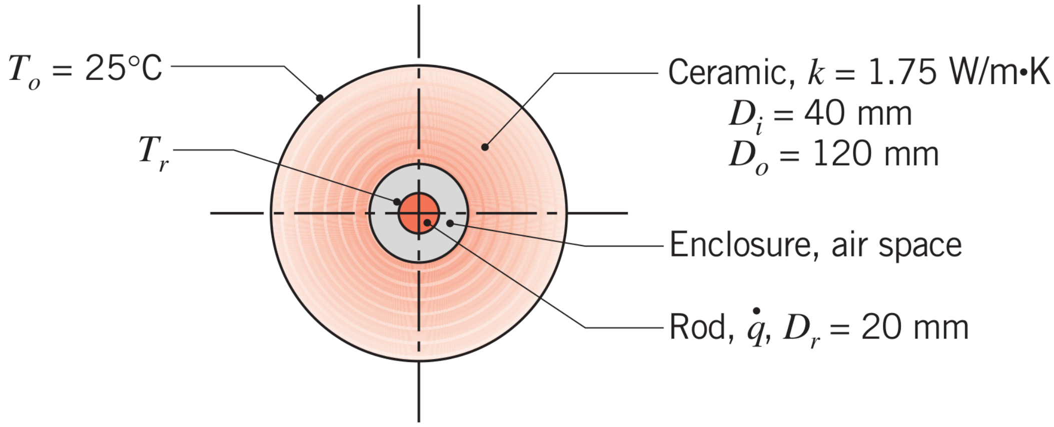


(a) Determine the thermal resistance between the surface temperature and the sensing temperature.

(b) If the sensing temperature is $T_{\text{sen}} = 28.5\ °\text{C}$, determine the surface temperature.

Ignore nanoscale heat transfer effects.

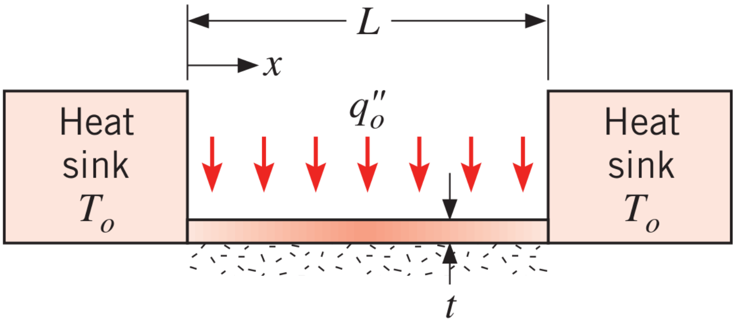
3. Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of $\dot{q} = 2 \times 10^6$ W/m^3. The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air.



The thermal resistance per unit length due to radiation between the enclosure surfaces is $R_{\text{rad}} = 0.30$ m $\cdot$ K/W, and the coefficient associated with free convection in the enclosure is $h = 20$ W/m$^2 \cdot$ K.

- Construct a thermal circuit that can be used to calculate the surface temperature of the rod, $T(r)$. Label all temperatures, heat rates, and thermal resistances, and evaluate each thermal resistance.
- Calculate the surface temperature of the rod for the prescribed conditions.

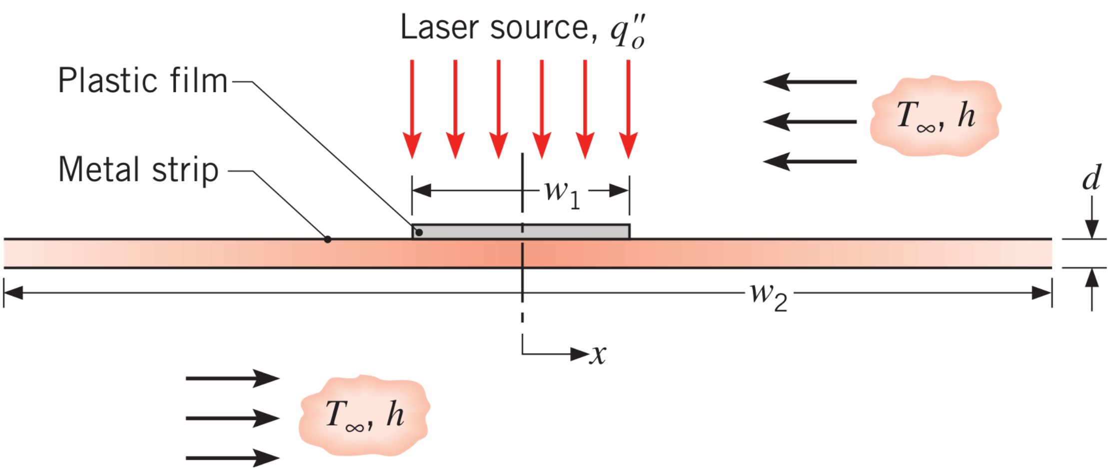
4. A thin flat plate of length L, thickness t, and width $W >> L$ is thermally joined to two large heat sinks that are maintained at a temperature T_0. The bottom of the plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q''_0



   (a) Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

   (b) Now, suppose the heat sinks are at different temperatures, $T(0) = T_0$ and $T(L) = T_L$, and the bottom surface is no longer insulated. Convection heat transfer is now allowed to occur between this surface and a fluid at T_∞, with a convection coefficient h. Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

5. A bonding operation utilizes a laser to provide a constant heat flux, q''_0 , across the top surface of a thin adhesive-backed, plastic film to be affixed to a metal strip as shown in the sketch. The metal strip has a thickness $d = 1.25$ mm, and its width is large relative to that of the film. The thermophysical properties of the strip are $\rho = 7850\ \mathrm{kg/m^3}$, $c_p = 435\ \mathrm{J/kg \cdot K}$, and $k = 60\ \mathrm{W/m \cdot K}$. The thermal resistance of the plastic film of width $w_1 = 40$ mm is negligible. The upper and lower surfaces of the strip (including the plastic film) experience convection with air at 25 °C and a convection coefficient of 10 $\mathrm{W/(m^2K)}$. The strip and film are very long in the direction normal to the page. Assume the edges of the metal strip are at the air temperature (T_∞).



(a) Derive an expression for the temperature distribution in the portion of the steel strip with the plastic film.

(b) If the heat flux provided by the laser is 10 $\mathrm{kW/m^2}$, determine the temperature of the plastic film at the center ($x = 0$) and its edges ($x = \pm w_1/2$).

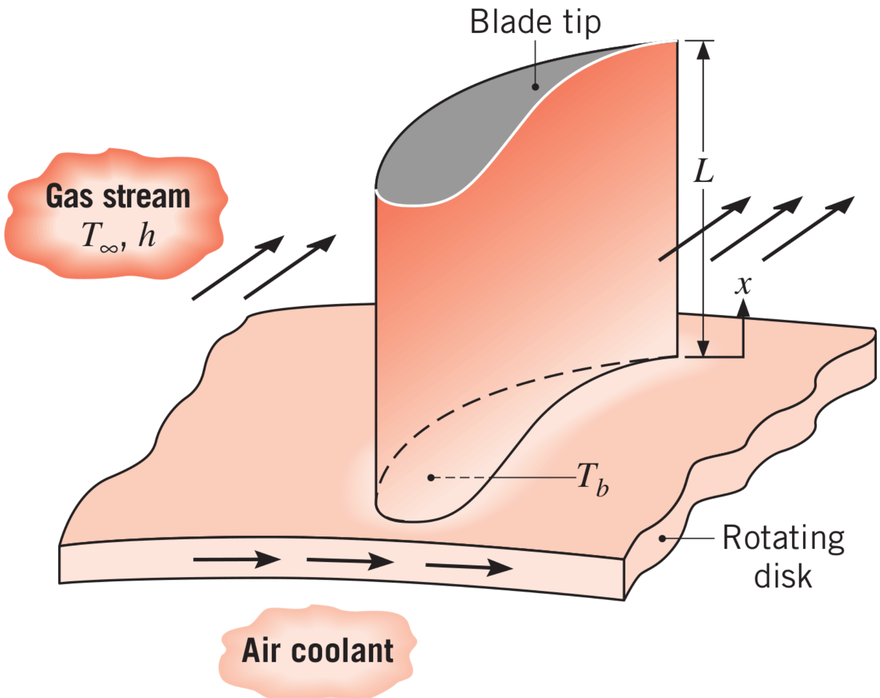
(c) Plot the temperature distribution for the entire strip and point out its special features.

6. Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200$ °C and maintains a convection coefficient of $h = 250$ W/(m^{2}K) over the blade.

   The blades, which are fabricated from Inconel, $k = 20$ W/(mK), have a length of $L = 50$ mm. The blade profile has a uniform cross-sectional area of $A_c = 6 \times 10^{-4}$ m^2 and a perimeter of $P = 110$ mm. A proposed blade-cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature of $T_b = 300$ °C.

   (a) If the maximum allowable blade temperature is 1050 °C and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?

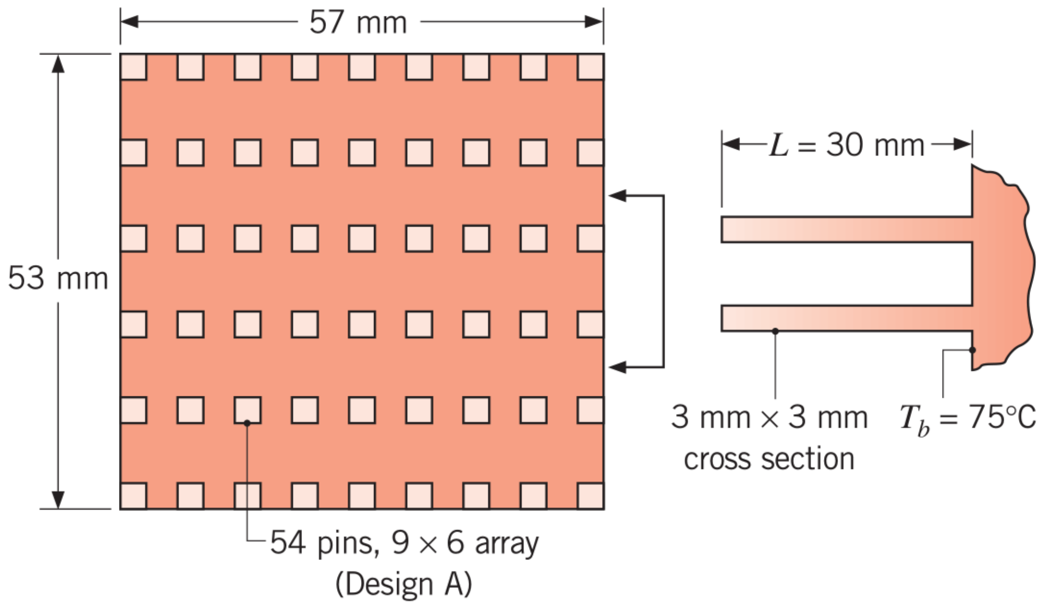
   (b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?



7. Because of the large number of devices in today's PC chips, finned heat sinks are often used to maintain the chip at an acceptable operating temperature. Two fin designs are to be evaluated, both of which have base (unfinned) area dimensions of 53 mm × 57 mm. The fins are of square cross section and fabricated from an extruded aluminum alloy with a thermal conductivity of 175 W/(mK). Cooling air may be supplied at 25 °C, and the maximum allowable chip temperature is 75 °C. Other features of the design and operating conditions are tabulated.

Design	Cross section (mm × mm)	Length (mm)	Number of fins	h (W/(m^{2}K))
A	3 × 3	30	6 × 9	125
B	1 × 1	7	14 × 17	375

Determine which fin arrangement is superior. In your analysis, calculate the heat rate, efficiency, and effectiveness of a single fin, as well as the total heat rate and overall efficiency of the array. Since real estate inside the computer enclosure is important, compare the total heat rate per unit volume for the two designs.



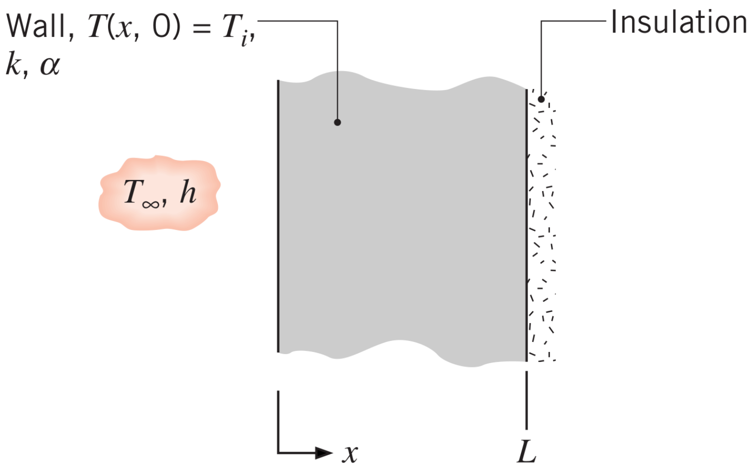
8. Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20$ W/(m^{2}K). Assuming the properties of the steel to be $k = 40$ W/(mK), $\rho = 7800$ kg/m^3, and $c = 600$ J/(kgK), estimate the time required for the cooling process.

9. Consider the steel balls of Problem 8, except now the air temperature increases with time as $T_\infty(t) = 325\text{ K} + at$, where $a = 0.1875$ K/s.

   (a) Sketch the ball temperature versus time for $0 \leq t \leq 1$ h. Also show the ambient temperature, T_∞ , in your graph. Explain special features of the ball temperature behaviour.

   (b) Find an expression for the ball temperature as a function of time $T(t)$, and plot the ball temperature for $0 \leq t \leq 1$ h. Was your sketch correct?

10. Consider the one-dimensional wall shown in the sketch, which is initially at a uniform temperature T_i and is suddenly subjected to the convection boundary condition with a fluid at T_∞.



For a particular wall, case 1, the temperature at $x = L_1$ after $t_1 = 100$ s is $T_1(L_1, t_1) = 315\,^\circ C$. Another wall, case 2, has different thickness and thermal conditions as shown.

Case	L (m)	α (m^2/s)	k (W/(mK))	T_i (°C)	T_∞ (°C)	h (W/(m^{2}K))
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

How long will it take for the second wall to reach 28.5 °C at the position $x = L_2$? Use as the basis for analysis, the dimensionless functional dependence for the transient temperature distribution expressed in Equation 5.41.

11. Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830$ kg/m^3 , $c = 550$ J/(kgK), $k = 48$ W/(mK)), which is initially at a uniform temperature of $T_i = 200$ °C and is to be heated to a minimum temperature of 550 °C. Heating is effected in a gas-fired furnace, where products of combustion at $T_\infty = 800$ °C maintain a convection coefficient of $h =$ 250 W/(m^{2}K) on both surfaces of the plate. How long should the plate be left in the furnace?

12. A long cylinder of 30-mm diameter, initially at a uniform temperature of 1000 K, is suddenly quenched in a large, constant-temperature oil bath at 350 K. The cylinder properties are $k = 1.7$ W/(mK), $c = 1600$ J/(kgK), and $\rho = 400$ kg/m^3, while the convection coefficient is 50 W/(m^{2}K). Calculate the time required for the surface of the cylinder to reach 500 K.

13. A thick steel slab ($\rho = 7800$ kg/m^{3}3 , $c = 480$ J/(kgK), $k = 50$ W/(mK)) is initially at 300 °C and is cooled by water jets impinging on one of its surfaces. The temperature of the water is 25 °C, and the jets maintain an extremely large, approximately uniform convection coefficient at the surface. Assuming that the surface is maintained at the temperature of the water throughout the cooling, how long will it take for the temperature to reach 50 °C at a distance of 25 mm from the surface?

14. A simple procedure for measuring surface convection heat transfer coefficients involves coating the surface with a thin layer of material having a precise melting point temperature. The surface is then heated and, by determining the time required for melting to occur, the convection coefficient is determined. The following experimental arrangement uses the procedure to determine the convection coefficient for gas flow normal to a surface. Specifically, a long copper rod is encased in a super insulator of very low thermal conductivity, and a very thin coating is applied to its exposed surface.

    If the rod is initially at 25 °C and gas flow for which $h = 200$ W/(m^{2}K) and $T_\infty = 300$ °C is initiated, what is the melting point temperature of the coating if melting is observed to occur at $t = 400$ s?

15. A semi-infinite aluminum cylinder ($k = 237$ W/(mK), $\alpha = 9.71 \times 10^{-5}$ m^2/s) of diameter $D = 15$ cm is initially at a uniform temperature of $T_i = 150$ °C. The cylinder is now placed in water at 10 °C, where heat transfer takes place by convection with a heat transfer coefficient of $h = 140$ W/(m^{2}K). Determine the temperature at the center of the cylinder 5 cm from the end surface 8 min after the start of cooling.